RESEARCH ARTICLE

Mixing at junctions in water distribution systems: an experimental study

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ABSTRACT
The accurate prediction of solute transport in water distribution systems has become a critical component of efforts aimed at delivering clean drinking water safely to large urban populations. One of the central assumptions that water quality models have traditionally relied upon dictates that, in a four-way junction, two incoming flows of differing quality will mix perfectly to produce two outgoing flows of equal quality, and that such complete mixing occurs irrespective of the specific assemblage characteristics of any particular four-way junction. In this study, laboratory experiments were conducted in order to characterize solute mixing patterns at double-tee and wye junctions, both of which are commonly found in urban water distribution systems. Results show that mixing at double-tee junctions tends to be less than complete when the tee connectors are located adjacent to each other and mix to a greater degree than flows at cross junctions.

Introduction

Accurately predicting water quality within pipe networks involves a number of technical challenges. Most large networks, for instance, typically distribute water drawn from more than one source and treated at different locations. Furthermore, water quality predictions must be reliable because they are being used to develop real-time monitoring capabilities on which early-warning systems depend, especially when sensor stations must be strategically placed to help detect accidental and deliberate intrusions of contaminants. The rate at which two water flows of differing quality will mix when they converge at a junction has received considerable attention in the past decade because water distribution systems serving large municipal areas invariably include junctions at which three or four pipes branch out or converge from different parts of the pipe network system. The majority of these connectors can be represented with sufficient accuracy by the basic set of configurations shown in Figure 1, where the pipe legs are labeled according to their location as W, S, N, and E (or E, in those configurations where the west and east pipe legs are located on the same side of the junction). Cross junctions, for instance (Figure 1(a) and (b)), refer to two closely located tee connectors (i.e. double-tee 'N' junctions) facing opposite directions (Figure 1(c) and (d)), two single-tee junctions aligned face the same direction to form a U shape (Figure 1(e) and (f)), and wye junctions consist of a tee connector and a 45° connector placed at a distance (Figure 1(g) and (h)).

The water quality models used to analyze water distribution systems have assumed an instantaneous and complete mixing of biological or chemical agents at junctions. However, recent studies have shown that solute mixing at cross junctions is far from being complete (van Bloemen Waanders et al. 2005; Romero-Gomez et al. 2006; McKenna et al. 2007). These studies explored solute mixing as it occurs at cross junctions that involve a variety of pipe diameters and flow conditions. In a more focused experiment based on inflow and outflow ratios under a wide range of flow conditions, Austin et al. (2008) was able to characterize mixing patterns at a cross junction. Using computational fluid dynamics (CFD), Romero-Gomez, Ho, and Choi (2008) and Ho, Choi, and McKenna (2007) used the Reynolds averaged Navier-Stokes (RANS) turbulence model and Webb (2007) used the large eddy simulation model (LES) to study mixing mechanisms at cross junctions. These studies found evidence that mixing is in fact incomplete, and Song et al. (2009) showed that assuming complete mixing generated considerable modeling errors in laboratory-scale water pipe networks that include multiple cross junctions. They concluded that detailed characterizations of mixing phenomena at various types of pipe junctions could significantly improve model accuracy in predicting the transport of constituents in water distribution systems. Shao et al. (2014) examined, experimentally and numerically, the mixing at cross and double-tee junctions and developed an analytical formula as a function of the flow ratio between the two opposing branches.

Studies involving applications other than water distribution networks have also uncovered evidence of incomplete mixing. Kok and van del Wal (1996) analyzed the mixing of two gasses in a single-tee junction mixer using the k-ε turbulent model and compared the model results with experimental results. Brucker (1997) studied three-dimensional flow in a single-tee junction using particle image velocimetry and focusing on the interaction...
of the jet and the cross flow. Plesniak and Cusano (2005) conducted a comprehensive experimental investigation of a type of confined rectangular jet (in a cross flow) that occurs in a variety of manufacturing processes, and showed that the flow does not necessarily develop symmetrically. In summary, the evidence uncovered by all these studies invariably suggests that simplifying double-tee and wye-type configurations as cross junctions may lead to considerable deviations in modeled water quality, and that assuming complete mixing in all instances may further amplify the discrepancies.

To improve the accuracy of water quality prediction in water pipe networks that include different types of junctions, the solute mixing patterns associated with the different junction geometries must be characterized. Hence, this study was designed to achieve two of these characterizations—the mixing at double-tee (N/U) and wye (YN/YU) junctions—under different flow scenarios. To the best of our knowledge, this work contributes to broaden the understanding of mixing phenomena at pipe junctions because previous efforts to enhance water quality predictions in water distribution systems did not characterize the junction configurations, and instead simplified them as cross junctions. In addition, this work produced a data set upon which solute concentration predictions in large-scale systems can be calculated. In fact, the resulting data were later implemented in a modified code (AZRED v2.0, 2017). This new code was applied to an exemplary pipe network in order to compare the transport patterns of a solute passing through the various junction configurations.

**Material and methods**

**Definition and scenarios**

The spacing between the fittings is described in terms of the dimensionless length (Figure 1):

\[ L^* = \frac{L}{D} \]  \hspace{1cm} (1)

where \( L^* \) is the dimensionless distance between the two fittings, \( L \) is the distance between fittings, and \( D \) is the inner pipe diameter. In all instances, all four pipes have the same diameter and the ratios of the inlet and outlet flow rates are described as a ratio of the Reynolds number:

\[ R_i = \frac{\rho DU_i}{\mu} \]  \hspace{1cm} (2)

where \( i \) is the pipe leg in N, E, S, or W directions, \( U \) is the average flow velocity, \( \rho \) is the density of water, and \( \mu \) is the dynamic viscosity. The average Reynolds number of the inlet and outlet pipes is defined as:

\[ R_{\text{ave}} = \frac{R_N + R_E + R_S + R_W}{4} \]  \hspace{1cm} (3)

The ratio of inlet Reynolds numbers is defined as:

\[ R_{5/1} = \frac{R_N}{R_W} \]  \hspace{1cm} (4)

The inflow configuration was represented in terms of a normalized flow rather than the Reynolds number ratio as given above. The normalized inflow from the south inlet is defined as:

\[ Q_s^* = \frac{Q_s}{Q_s + Q_w} \]  \hspace{1cm} (5)

where \( Q_s \) and \( Q_w \) are the flows at the south and west inlets, respectively. The ratio of outlet Reynolds numbers is defined as:

\[ R_{E/W} = \frac{R_{E\text{ or }E^{-}}}{R_{W\text{ or }W^{-}}} \]  \hspace{1cm} (6)

As with the inflows, the outflow configuration is described through a normalized flow (at the east outlet in this case) rather than the Reynolds number ratio. This is defined as:

\[ Q_e^* = \frac{Q_e}{Q_s + Q_w} \]  \hspace{1cm} (7)

where \( Q_e^* \) is the normalized flow of the east outlet, and \( Q_e \) and \( Q_w \) are the flows at the east and north outlets, respectively. The east pipe outlet is referred to as E- in those junctions shown in Figure 1 when the west and east pipes are located on the same side of
the junction, that is, in the U1 and U2 type of junctions, as well as in the YU type.

To achieve an accurate understanding of the fundamental nature of mixing phenomena at the various mixing junctions, three scenarios were investigated:

- **Scenario 1**: $Q_s = Q_{sw} = Q_w = Q_{\text{inlet}}$. All four flows in the system remained equal to one another and $R_{\text{avg}}$ was used as the independent variable.
- **Scenario 2**: $Q_s \neq Q_{sw} = Q_w = Q_{\text{inlet}}$. The outflows remained equal ($Q_{\text{inlet}}^* = 0.5$), and the inflows were described using the normalized south inlet flow $Q_{\text{in}}^*$. $R_{\text{avg}}$ was also held constant at 26,000.
- **Scenario 3**: $Q_s = Q_{sw} = Q_w = Q_{\text{inlet}}^*$. The inflows remained equal ($Q_{\text{inlet}}^* = 0.5$), and the outflows varied ($R_{\text{inlet}}$ or $R_{\text{inlet}}/R_{\text{avg}}$). $R_{\text{avg}}$ remained at 26,000.

Mixing at junctions is presented in terms of the dimensionless concentration at the east outlet ($C_e^*$ or $C_w^*$, depending on the type of junction), which is defined as:

$$C_e^* = \frac{C_e - C_w}{C_s - C_w} \quad (8a)$$

or

$$C_w^* = \frac{C_e - C_w}{C_s - C_w} \quad (8b)$$

where $C$ is the solute concentration at each pipe.

**Experimental setup**

Experiments were conducted in the Water Distribution Network Laboratory of the Water Village at the University of Arizona, Tucson, AZ. The experimental setup consisted of two pumps, two frequency controllers, a fresh water tank, another water tank at high NaCl concentration, four gate valves, and a set of junctions as shown in Figure 1. More details of the experimental setup can be found in Austin et al. (2008). The present junction configurations were created by connecting two commercially available PVC fittings (NIBCO Inc., Elkhart, IN, and Spears Inc., Sylmar, CA). As presented in Figures 2(b–d), water was pumped from two separate tanks into the fitting and then drained through two exit pipes. The $R_{\text{avg}}$ was maintained at 26,000 throughout the test to allow for comparisons between the present results and those for cross junctions in Austin et al. (2008) and also to maintain a turbulent flow regime. However, the experiments in Scenario 1 were conducted at a series of Reynolds numbers in order to determine the solute mixing rate’s sensitivity to the flow regime.

Flow meter and conductivity sensors were placed at each inflow and outflow pipe leg to record flow rates and solute concentration, respectively. The flow sensors were of a paddle wheel design (FP-5600 and manufactured by the Omega Corporation, Stamford, CT), while NaCl concentration was measured in electrical conductivity using four-ring potentiometric probes and transmitters (CDE-1201, CDTX-1203, the Omega Corporation, Stamford, CT). Electrical conductivity readings were then converted into concentration values of sodium chloride by means of a second-order polynomial. A dimensionless concentration was thereafter calculated in order to describe the level of mixing taking place at the junction. Data were collected with a data logger (CR3000, Campbell Scientific, Logan, UT) that also controlled the pumps by relying on feedback from sensors and under the command of the variable-frequency drive pump controllers. The repeatability of the flow sensors is 0.5% of the full range (0.36 LPM), with a range of 1.34 to 71.9 LPM. Each of the sensors is calibrated by collecting water discharged through the sensor at a constant rate and timing the collection. The accuracy of the conductivity sensors as described by the manufacturer is ±2% of the full range (2 mS/cm).

All tests were run until the solute concentration reached a steady state; data were then collected over a 60-s period. This protocol reduced the chances that signal noise or unsteady mixing patterns at the junction would have any significant effect. Calibration of both sensors was performed regularly in accordance with the laboratory QA/QC manual. Flow and NaCl mass balances were also evaluated to ensure rigorous experimental processes and data sets. All runs were repeated three times for a given flow configuration, and averaged values are herein presented. Averaging the ranges from all flow configurations tested in scenario 2, for example, the variability in the ratio of inlet Reynolds numbers ($R_{\text{inlet}}$, equation 4) was 0.0052 and in dimensionless concentrations ($C^*$, equation 8a) it was approximately 2%.

**Results and discussion**

**Scenario 1: equal Inflows and outflows**

The dimensionless solute concentrations at the junction outlet demonstrated very small variations after the flow rate reached the turbulent flow regime ($R > 10,000$) (Figure 3). Hence, it was assumed that solute mixing occurred independently of flow rate in the turbulent regime. Thus an averaged value of the dimensionless concentrations for $R < 10,000$ was taken as a representative mixing value for each configuration.

**Scenario 2: equal outflows, varying inflows**

The dimensionless concentrations recorded at the east outlets (E or $E^*$) of the general double-tee cases in scenario 2 are presented in Figure 4. The overall trend that occurred at the double-tee junction (and the mixing patterns as well) resembled that of the mixing at cross junctions; that is, the dimensionless concentration values increase as the $Q_e^*$ ratio increases. Asymptotically, the dimensionless concentration should approach zero as $Q_e^* \to 0$, whereas it should approach 1 as $Q_e^* \to 1$ since all outlet flows come from either the west leg (clean water) or the south leg (solute water), respectively. Similar to the trends in scenario 1, the double-tee junction enhanced solute mixing (compared to mixing at a cross junction), and the trend towards complete mixing was observed with increasing values of $L^*$. Cross junctions involving two opposing flows (X2) produced essentially complete mixing results, thereby behaving in a manner that was fundamentally different from that found during the X1-type experiments. Romero-Gomez et al. (2008) reported that in X1 junctions, higher rates of momentum transfer in the mixing zone (represented by the eddy viscosity in the turbulence models) gave rise to greater scalar mixing. In comparison to the X1-type
Figure 2. (a) Schematic drawing of experimental setup for mixing at a wye junction, and various configurations which have been tested: (b) N junction, (c) U junction (c) and (d) YN/YU junction.

Figure 3. Solute mixing at the double-tee junction for scenario 1. C_1^* or C_2^* indicate dimensionless solute concentration of the E or E- leg flow in Figure 1. L^* represents the diameter distance between two single-Tees (Equation 1), whereas N and U indicate N and U junction configurations (see Figure 1). The cross mixing data for the XI configuration are obtained from Austin et al. (2008). Junction, the two impinging flows in scenario X2 enhanced the momentum transfer rate and ultimately yielded complete mixing rates.

Figure 4. Solute mixing at the double-tee junctions for scenario 1.

Figure 5 shows the dimensionless concentrations at the E or E- outlets of scenario 2 as a function of Q^*_E, at two special N and U junctions (YN and YU), and the results obtained from both complete mixing and bulk mixing are also presented to provide a better understanding of the level of mixing taking place. Ho et al. (2007) suggested that bulk mixing could be assumed as a lower boundary representing complete bifurcation of flow in
fittings. It should be noted that these points were calculated in accordance with the analytical relationships proposed by Ho et al. (2007), which are mainly governed by the inlet condition. As $Q^{*}_{S} \rightarrow 0$ there is no inflow from the tracer source, giving a $C_{S}^{*}$ or $C_{T}^{*}$ value of 0. On the other hand, as $Q^{*}_{S} \rightarrow 1$, all of the flow entering the junction comes from the tracer tank, giving a $C_{S}^{*}$ or $C_{T}^{*}$ value of 1.

A comparison of Figure 4 to Figure 5 shows that the results obtained from the case of YN-type junctions were very similar to the N-type junctions in that, as the distance between the fittings increases, flow was allowed to develop to a larger extent, which enhances the solute transfer towards complete mixing. But that similarity did not manifest between the U-type and the YU-type junctions. In the YU junctions, an $L^{*}$ distance of 2.5 actually produced greater mixing rates than did larger $L^{*}$ values to $Q^{*}_{S}$ levels up to 0.5 values.

**Scenario 3: equal inflows, varying outflows**

The results obtained from scenario 3 are presented in Figure 6 and Figure 7. Both complete mixing and bulk mixing rates are plotted in Figure 6 (which is similar to the presentation of the scenario-2 results). At $Q^{*}_{E} = 1$, $C_{S}^{*}$ or $C_{T}^{*}$ were equal to 0.5 because such an instance collects the incoming flow and discharges it through a single outlet (the east outlet), giving rise to complete mixing. The mixing rates at YN- and N-type junctions follow similar trends in that they monotonically decreased with $Q^{*}_{E}$ and that they did so in equivalent magnitudes, which suggested that flow conditions took place in a similar manner in both types of junction. The larger $L^{*}$ values promoted the formation of fully developed conditions which in turn enhanced mixing in N1-, U1-, and YN-type of junctions. In contrast, YU-type junctions yielded nearly complete mixing at small $L^{*}$ values rather than at larger $L^{*}$. Because of this mixing trend with $L^{*}$, this latter scenario deserves further examination by means of numerical models that can resolve localized flow conditions by means of large and detached eddy simulations.

**A sample network application**

In order to demonstrate the potential implications of the current study’s findings, an exemplary pipe network that included two four-way junctions (labeled as nodes A and B in Figure 8) and three demand points was set up. The network served to quantify the impact that the empirical mixing rates developed in this study would have on the overall water quality at all nodes in the system. Two reservoirs supplied the network, one with a high tracer concentration of 600 mg/L. Steady-state simulations of the pipe flows and water quality throughout the network were conducted, one model assuming a complete mixing and the other an incomplete mixing. The software EPANET was used to determine the Reynolds numbers of the pipe flows (Table 1).

Incomplete mixing transport at double-tee junctions was estimated based on experimental data. The tracer concentrations at the three demand points (D1, D2, and D3 in Figure 8) are presented in Table 2. The error percentage indicates the margin of error in predicting solute concentration when using the complete mixing assumption as compared with that when using the incomplete mixing model. Whereas the incomplete mixing results can be derived from crosses and double-tee junctions at three different $L^{*}$ values, the type of junction becomes irrelevant under complete mixing assumptions. As shown in Table 2, the complete mixing model generates an error in solute concentration estimation of up to 248% when applied to the exemplary pipe network equipped with cross junctions. This error percentage was greatly reduced when mixing rates at double-tee junctions were used instead, but values of $E1$ can
still reach 42% in this simplified network. Increasing the $L^*$ value further reduced the percentage error, with the case of $L^* = 10$ showing values very close to those derived when relying on the complete mixing assumption (less than 6% error). Choi, Shen, and Austin (2008) indicated that the complete mixing model tended to yield more evenly distributed concentrations of tracer, whereas the incomplete mixing model showed a more directed solute plume spread.

**Conclusion**

The series of experiments conducted in this study investigated solute mixing that occurred at double-tee and wye junctions under different hydraulic scenarios. The double-tee junction exhibited trends of solute mixing similar to those of cross

**Disclosure statement**

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